Institut la centrale — Analyse 1 — Exercices proposés DL

Exercice 1 DLn de $\operatorname{sh}x$ et $\operatorname{ch}x$

On a $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ $\qquad e^{-x} = 1 - \frac{x}{2!} + \frac{x^2}{3!} - \frac{x^3}{3!} + \dots$

Alors $\operatorname{ch}x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + x^{2n}\varepsilon(x)$

et $\operatorname{sh}x = \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + x^{2n+1}\varepsilon(x)$

Exercice 2 a/ $DL_4(0)$ de $\frac{e^x}{1-x}$

$$\frac{e^x}{1-x} = e^x \times \frac{1}{1-x} = \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + x^4\varepsilon(x)\right)\left(1 + x + x^2 + x^3 + x^4 + x^4\varepsilon(x)\right)$$
$$= 1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \frac{65}{24}x^4 + x^4\varepsilon(x)$$

b/ $DL_4(0)$ de $\sin^2 x$

On a $\sin x = x - \frac{x^3}{6} + x^3\varepsilon(x)$ d'où $\sin^2 x = \left(x - \frac{x^3}{6} + x^3\varepsilon(x)\right)^2 = x^2 - \frac{x^4}{3} + x^4\varepsilon(x)$

Exercice 3 a/ $DL_5(0)$ de $\tan x$

1° méthode : $\tan x = \frac{\sin x}{\cos x}$ On effectue la division suivant les degrés croissants

$$\begin{array}{l|l} x - \frac{x^3}{6} + \frac{x^5}{120} + \dots & 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots \\ \hline -x + \frac{x^3}{2} - \frac{x^5}{24} & x + \frac{x^3}{3} + \frac{2x^5}{5} + \dots \\ \frac{x^3}{3} - \frac{x^5}{36} & \\ -\frac{x^3}{3} + \frac{x^5}{6} & \\ \hline \frac{2x^5}{5} & \end{array}$$

d'où $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{5} + x^5\varepsilon(x)$

2° méthode $\tan x = \frac{\sin x}{\cos x} = \frac{x - \frac{x^3}{6} + \frac{x^5}{120} + x^5\varepsilon(x)}{1 - \left(\frac{x^2}{2} - \frac{x^4}{24} + x^5\varepsilon(x)\right)}$

$$\tan x = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + x^5\varepsilon(x)\right)\left(1 + \left(\frac{x^2}{2} - \frac{x^4}{24}\right) + \left(\frac{x^2}{2} - \frac{x^4}{24}\right)^2 + x^5\varepsilon(x)\right)$$
$$= \left(x - \frac{x^3}{6} + \frac{x^5}{120} + x^5\varepsilon(x)\right)\left(1 + \frac{x^2}{2} - \frac{x^4}{24} + \frac{x^4}{4} + x^5\varepsilon(x)\right) = x + \frac{x^3}{3} + \frac{2x^5}{5} + x^5\varepsilon(x)$$

b/ $DL_4(0)$ de $x\operatorname{cotan}x$

$$x\operatorname{cotan}x = \frac{x}{\tan x} = \frac{x}{x + \frac{x^3}{3} + \frac{2x^5}{5} + x^5\varepsilon(x)} = \frac{1}{1 + \frac{x^2}{3} + \frac{2x^4}{5} + x^4\varepsilon(x)}$$
$$= 1 - \left(\frac{x^2}{3} + \frac{2x^4}{5}\right) + \left(\frac{x^2}{3} + \frac{2x^4}{5}\right)^2 + x^4\varepsilon(x) = 1 - \frac{x^2}{3} - \frac{2x^4}{5} + \frac{x^4}{9} + x^4\varepsilon(x)$$
$$= 1 - \frac{x^2}{3} - \frac{13}{45}x^4 + x^4\varepsilon(x)$$

Exercice 4 a/ $DL_4(0)$ de $h(x) = e^{1-\cos x}$

$$h(x) = e^{1-\cos x} = e^{\frac{x^2}{2} + \frac{x^4}{24} + x^4\varepsilon(x)} = 1 + \left(\frac{x^2}{2} + \frac{x^4}{24}\right) + \frac{\left(\frac{x^2}{2} + \frac{x^4}{24}\right)^2}{2} + x^4\varepsilon(x)$$
$$= 1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^4}{8} + x^4\varepsilon(x) = 1 + \frac{x^2}{2} + \frac{x^4}{6} + x^4\varepsilon(x)$$

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b/ $f(x) = \sqrt{\cos x} = \sqrt{1+\left(-\frac{x^2}{2}+\frac{x^4}{24}+x^4\varepsilon(x)\right)} = \left(1+\left(-\frac{x^2}{2}+\frac{x^4}{24}+x^4\varepsilon(x)\right)\right)^{1/2}$

$= 1+\frac{1}{2}\left(-\frac{x^2}{2}+\frac{x^4}{24}\right)+\frac{1/2(1/2-1)}{2}\left(-\frac{x^2}{2}+\frac{x^4}{24}\right)^2+x^4\varepsilon(x)$

$= 1-\frac{x^2}{4}+\frac{x^4}{48}-\frac{1}{8}\left(\frac{x^4}{4}\right)+x^4\varepsilon(x) = 1-\frac{x^2}{4}-\frac{x^4}{96}+x^4\varepsilon(x)$

c/ $g(x) = e^{\frac{1}{1-x}} = e^{1+x+x^2+x^3+x^4+x^4\varepsilon(x)} = e.e^{x+x^2+x^3+x^4+x^4\varepsilon(x)}$

$= e.\left(1+(x+x^2+x^3+x^4)+\frac{(x+x^2+x^3+x^4)^2}{2}+\frac{(x+x^2+x^3+x^4)^3}{6}+\frac{(x+x^2+x^3+x^4)^4}{24}+x^4\varepsilon(x)\right)$

$= e\left(1+x+x^2+x^3+x^4+\frac{1}{2}(x^2+x^4+2x^3+2x^4)+\frac{1}{6}(x^3+3x^4)+\frac{x^4}{24}+x^4\varepsilon(x)\right)$

$= e\left(1+x+\frac{3}{2}x^2+\frac{13}{6}x^3+\frac{73}{24}x^4+x^4\varepsilon(x)\right)$

<u>Exercice 5</u> a/ $DL_n(0)$: $f(x) = \ln(1+x)$

On a $f'(x) = \frac{1}{1+x} = 1-x+x^2-x^3+\dots+(-1)^{n-1}x^{n-1}+x^{n-1}\varepsilon(x)$

d'où $f(x) = \int_0^x \frac{1}{1+t}dt = x-\frac{x^2}{2}+\frac{x^3}{3}+\dots+(-1)^{n-1}\cdot\frac{x^n}{n}+x^n\varepsilon(x)$

b/ $DL_{2n}(0)$: $f(x) = \text{Arctan}\,x$

$f'(x) = \frac{1}{1+x^2} = 1-x^2+x^4-x^6+\dots+(-1)^{n-1}\cdot x^{2n-2}+x^{2n-2}\varepsilon(x)$

$f(x) = \int_0^x \frac{1}{1+t^2}dt = x-\frac{x^3}{3}+\frac{x^5}{5}+\dots+(-1)^{n-1}\cdot\frac{x^{2n-1}}{2n-1}+x^{2n-1}\varepsilon(x)$

c/ $DL_3(0)$: $\text{Arcsin}\,x$, $\text{Arccos}\,x$, $\frac{1}{(1+x)^2}$

$(\text{Arcsin}\,x)' = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-1/2} = 1+\frac{1}{2}x^2+x^2\varepsilon(x) \Rightarrow \text{Arcsin}\,x =$

$\text{Arcsin}\,x = \int_0^x \frac{1}{\sqrt{1-t^2}}dt = x+\frac{x^3}{6}+x^3\varepsilon(x)$

$(\text{Arccos}\,x)' = -\frac{1}{\sqrt{1-x^2}} = -1-\frac{1}{2}x^2+x^2\varepsilon(x)$

$\text{Arccos}\,x = \int_0^x -\frac{1}{\sqrt{1-x^2}}dx+\frac{\pi}{2} = \frac{\pi}{2}-x-\frac{x^3}{6}+x^3\varepsilon(x)$ $\quad(\text{Arccos}\,0 = \pi/2)$

$\frac{1}{(1+x)^2} = \frac{1}{1+(x^2+2x)} = 1-(x^2+2x)+(x^2+2x)^2-(x^2+2x)^3+x^3\varepsilon(x) = 1-2x+3x^2-4x^3+x^3\varepsilon(x)$

<u>Exercice 6</u> a/ $DL_3(1)$ de $f(x) = \frac{\ln x}{x^2}$ On pose $h = x-1$ alors $x = h+1$

et $f(x) = \frac{\ln(1+h)}{(1+h)^2} = \left(h-\frac{h^2}{2}+\frac{h^3}{3}+h^3\varepsilon(h)\right)(1-2h+3h^2-4h^3+h^3\varepsilon(h))$

$= h-\frac{5}{2}h^2+\frac{13}{3}h^3+h^3\varepsilon(h) = (x-1)-\frac{5}{2}(x-1)^2+\frac{13}{3}(x-1)^3+(x-1)^3\varepsilon(x-1)$

b/ On pose $h = \frac{1}{x}$ alors $f(x) = \frac{1/h\,e^h}{1/h-1} = \frac{e^h}{1-h} = 1+2h+\frac{5}{2}h^2+\frac{8}{3}h^3+\frac{65}{24}h^4+h^4\varepsilon(h)$

d'où $f(x) = 1+\frac{2}{x}+\frac{5}{2x^2}+\frac{8}{3x^3}+\frac{65}{24x^4}+\frac{1}{x^4}\varepsilon(1/x)$ //

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